

# Mathematical Model of Vehicle Dynamics for Drifting in Motor Sports

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**KEY WORDS:** Vehicle dynamics, Motion control, Dynamic model, Drifting, Traction force, Kinetic-friction [B1]

Drifting in motorsports utilizes traction force as centripetal force during cornering; therefore, modeling of the traction force is indispensable. In this study, we introduce a vehicle model enabling analytical calculations. Three findings have emerged: Firstly, a model was developed to accommodate slip angles beyond the small-angle approximation, incorporating a driving force represented by kinetic friction opposite to the direction of contact patch velocity. Secondly, comparisons between field measurements and calculated values of yaw rate and lateral acceleration confirmed that the model accurately reproduces actual vehicle behavior. Lastly, analysis of steady state drifting on four-wheel-drive vehicle using the proposed model revealed the conditions necessary for drifting with zero steering input.

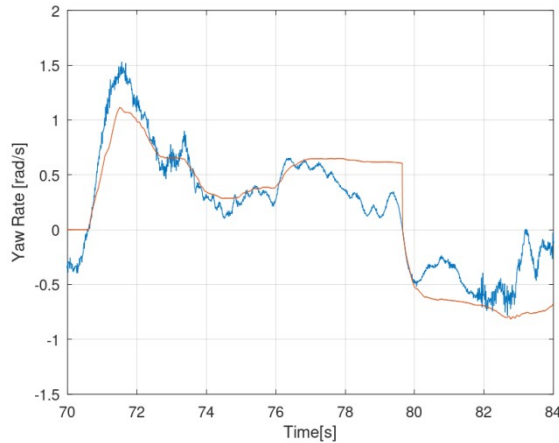
Equations of motion of steady state drifting on four-wheel-drive vehicle are as follows.

$$\text{Rotational: } I_z \ddot{\theta} = \frac{l_f l_r}{l_f + l_r} \mu' mg \left( \frac{\lambda_f \sin \delta - \eta_f \sin \rho}{\sqrt{\eta_f^2 + \lambda_f^2 - 2\eta_f \lambda_f \cos(\rho - \delta)}} + \frac{\eta_r \sin \sigma}{\sqrt{\eta_r^2 + \lambda_r^2 - 2\eta_r \lambda_r \cos \sigma}} \right)$$

$$\text{Lateral: } m v(\dot{\beta} + \dot{\theta}) = \frac{l_r}{l_f + l_r} \mu' mg \frac{\eta_f \sin(\beta - \rho) - \lambda_f \sin(\beta - \delta)}{\sqrt{\eta_f^2 + \lambda_f^2 - 2\eta_f \lambda_f \cos(\rho - \delta)}} + \frac{l_f}{l_f + l_r} \mu' mg \frac{\eta_r \sin(\beta - \sigma) - \lambda_r \sin \beta}{\sqrt{\eta_r^2 + \lambda_r^2 - 2\eta_r \lambda_r \cos \sigma}}$$

$$\text{Longitudinal: } m \dot{v} = \frac{l_r}{l_f + l_r} \mu' mg \frac{-\eta_f \cos(\beta - \rho) + \lambda_f \cos(\beta - \delta)}{\sqrt{\eta_f^2 + \lambda_f^2 - 2\eta_f \lambda_f \cos(\rho - \delta)}} + \frac{l_f}{l_f + l_r} \mu' mg \frac{-\eta_r \cos(\beta - \sigma) + \lambda_r \cos \beta}{\sqrt{\eta_r^2 + \lambda_r^2 - 2\eta_r \lambda_r \cos \sigma}}$$

The measurement and simulation results for vehicle dynamics are shown below. The section from 72 to 78 seconds represents the drifting phase. Although these dynamics fluctuate depending on the steering angle and the tire slip ratio caused by the accelerator opening, it was confirmed that the measurement and calculation results show the same trend.



The steering angle of a four-wheel drive vehicle during steady state drifting is given by the following equation, from which two conclusions can be drawn.

$$\delta = \frac{l_f + l_r}{r} \cos \beta \operatorname{sgn} \dot{\theta}$$

1. The same as the direction of the curve.
2. Approaches zero as the cornering radius or the slip angle increases.